

# KINEMATICS

Page 1

## VECTORS:

1. Electric current has magnitude as well as direction; still it is not a vector quantity. As its addition does not follow vector rules of addition.
2. Vectors which act along the axis of rotation are called axial vectors. For example angular velocity, torque etc.
3. Division of vector quantity is not permissible.
4. The magnitude of resultant **R** of two vectors **A** and **B** follows that

$$| \mathbf{A} - \mathbf{B} | \leq \mathbf{R} \leq | \mathbf{A} + \mathbf{B} |$$

5. The angle between two vectors is the smallest angle, when the two vectors are placed co-initial.
6.  $\hat{i} + \hat{j} + \hat{k}$  makes an angle of  $54.74^\circ$  with X, Y and Z axes.
7. *A vector can be resolved into infinite number of components but the maximum number of rectangular component is three.*
8. When three vectors **A**, **B** and **C** add-up to give zero resultant, then  $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$

## DOT AND CROSS PRODUCT OF VECTORS:

9.  $\mathbf{A} \cdot \mathbf{A} = A^2$
10. When  $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$ ,  $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{C} = \mathbf{C} \times \mathbf{A}$
11. A unit vector perpendicular to the plane containing A and B is  $\hat{n} = \frac{\mathbf{A} \times \mathbf{B}}{AB \sin \theta}$
12.  $| \mathbf{A} \times \mathbf{B} |$  = area of parallelogram formed by two vectors **A** and **B**  
= 2 X area of triangle formed by the two vectors  
=  $\frac{1}{2} | \mathbf{D}_1 \times \mathbf{D}_2 |$  where **D**<sub>1</sub> and **D**<sub>2</sub> are the diagonals of the  
parallelogram formed by the two vectors **A** and **B**.
13. Component of vector A along another vector B is  $A \cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{B}$
14. Two vectors are perpendicular when  $\mathbf{A} \cdot \mathbf{B} = 0$  and parallel if  $\mathbf{A} \times \mathbf{B} = 0$

## ONE DIMENSIONAL MOTION

15. For 1-D motion, angle between velocity and acceleration should be  $0^\circ$  or  $180^\circ$ .
16. (a)  $\frac{\text{Displacement}}{\text{distance}} \leq 1$  (b)  $\frac{\text{Average speed}}{\text{Average velocity}} \geq 1$
17. | Instantaneous velocity | = Instantaneous speed
18. If a body travels the first half distance with speed  $v_1$  and the second half distance with speed  $v_2$ , then the average speed of the body is

$$V_{\text{ave}} = \frac{2v_1v_2}{v_1+v_2}$$

19. If a body travels with a speed  $v_1$  for time  $t$  and then with speed  $v_2$  for another time  $t$ , then average speed

$$V_{\text{ave}} = \frac{v_1+v_2}{2}$$

20.  $a = v \frac{dv}{ds}$  and  $a = \frac{v-u}{t}$

21. Use equation of motion  $v = u + at$ ,  $S = ut + \frac{1}{2}at^2$  and  $v^2 - u^2 = 2as$ .

For  $S_n = u + \frac{a}{2}(2n-1)$

22. When a body is dropped from a height  $h$ , the velocity with which it falls is  $\sqrt{2gh}$  and the time taken is  $t = \sqrt{2h/g}$ .
23. When a body is thrown vertically upwards with a speed  $u$ , then the height attained by the body is  $h = \frac{u^2}{2g}$  and the time to reach this height is  $t' = \frac{u}{g}$

## TWO DIMENSIONAL MOTIONS

24.  $V_x = x/t$ ,  $v_y = y/t$ ,  $v_z = z/t$

25. If  $x = f(t)$  then  $V_x = \frac{dx}{dt}$ , similarly,  $v_y = \frac{dy}{dt}$ ,  $v_z = \frac{dz}{dt}$

26. The magnitude of velocity is given by  $V = \sqrt{V_x^2 + v_y^2 + v_z^2}$

27. Relative velocity  $\mathbf{V}_{AB} = \mathbf{V}_A - \mathbf{V}_B = \mathbf{V}_{AG} - \mathbf{V}_{BG}$ , where  $\mathbf{V}_{AG}$  or  $\mathbf{V}_A$  = velocity of A with respect to ground (G) and  $\mathbf{V}_{BG}$  or  $\mathbf{V}_B$  = velocity of B with respect to ground (G).

## FREELY FALLING BODIES

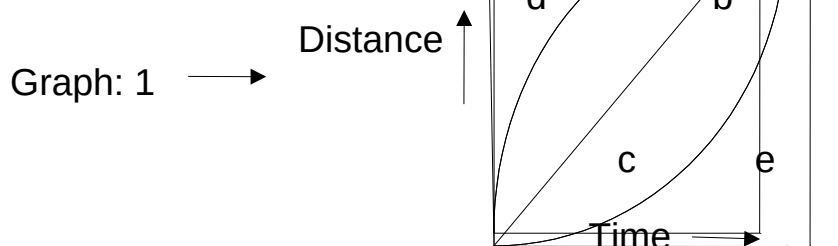
- If a body is dropped, then its velocity after travelling a distance  $h$  is  $\sqrt{2gh}$  and time is  $\sqrt{2h/g}$ . The ratio of the distances travelled after time  $t_1, t_2, t_3, \dots$  is  $t_1^2 : t_2^2 : t_3^2$ .

**PROJECTILE MOTION**

- $Y = x \tan\theta - (gx^2) / 2u^2 \cos^2\theta = x \tan\theta (1 - \frac{x}{R})$
- When a projectile is thrown vertically upwards, then the maximum height attained and time of flight is maximum, but the range is zero.
- When a projectile is thrown at an angle of  $45^\circ$ , then the range is maximum. Also in this case,  $R_{\max} = 4 H$ . If  $K$  is the kinetic energy with which the projectile is thrown, then the KE at the topmost point is  $K/2$  and Potential energy is also  $K/2$ .
- When a projectile is thrown two times with the same velocity, with angles of projection  $\theta$  and  $90 - \theta$ , then the range is same. If  $T_1$  and  $T_2$  are the time of flights, then  $T_1.T_2 = 2R/g$ .
- In non-uniform circular motion, the net acceleration  $a = \sqrt{a_r^2 + a_t^2}$ , where  $a_r$  &  $a_t$  represent radial and tangential components of acceleration. For uniform circular motion  $a_t = 0$ .
- In uniform circular motion velocity/acceleration/momentum change continuously but angular momentum remains conserved.
- $\mathbf{V} = \boldsymbol{\omega} \times \mathbf{r}$ ;  $\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$ ;  $\mathbf{a}_c = \boldsymbol{\omega} \times \mathbf{v}$
- $\frac{dI}{dt}$  Represent rate of change of speed but  $\left| \frac{dv}{dx} \right|$  represent the magnitude of acceleration.
- For two angles of projection  $\theta$  and  $90 - \theta$ , the range remains the same and the following relations hold good :
  - (a)  $R_1 / R_2 = 1$     (b)  $T_1 / T_2 = \tan\theta$     (c)  $H_1 / H_2 = \tan^2\theta$     (d)  $T_1 \times T_2 = 2R/g$
  - (e)  $H_1 \times H_2 = R^2/16$

**TYPE OF MOTION DEPENDS ON ACCELERATION:**

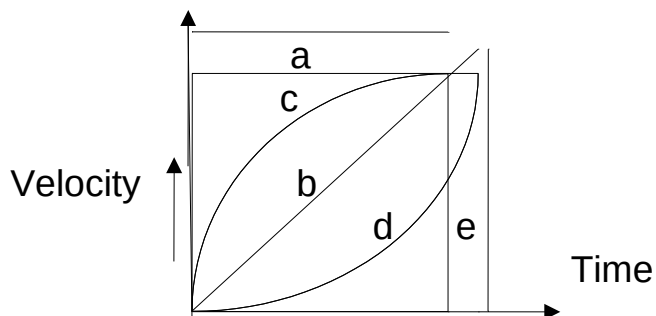
Type of motion	Centripetal acceleration	Tangential Acceleration
Uniform translator motion	$a_c = 0$	$a_t = 0$
Translatory motion with acceleration	$a_c = 0$	$a_t \neq 0$
Uniform circular motion	$a_c \neq 0$	$a_t = 0$
Non-uniform circular motion	$a_c \neq 0$	$a_t \neq 0$

**POSITION - TIME GRAPH**

<b><u>Graph</u></b>	<b><u>Angle with Time axis</u></b>	<b><u>velocity</u></b>	<b><u>Nature Of motion</u></b>
Graph (a)	$0^\circ$	0	Particle at Rest
Graph (b)	$90^\circ$	infinite	Impossible
Graph (c)	constant	constant	Uniform velocity
Graph (d)	$\theta$ increasing	increasing	Uniform acceleration
Graph (e)	$\theta$ decreasing	decreasing	Uniform acceleration

**VELOCITY-TIME GRAPH**

<b><u>Graph</u></b>	<b><u>Angle with Time axis</u></b>	<b><u>velocity</u></b>	<b><u>Nature Of motion</u></b>
Graph (a)	$0^\circ$	0	Uniform velocity
Graph (b)	$90^\circ$	infinite	Impossible
Graph (c)	constant	constant	Uniform acceleration
Graph (d)	$\theta$ increasing	increasing	Increasing acceleration
Graph (e)	$\theta$ decreasing	decreasing	Decreasing acceleration



## MULTIPLE CHOICE QUESTIONS

1. During projectile motion, the quantities that remain unchanged are:  
(a) force and vertical velocity      (b) acceleration and horizontal velocity  
(c) kinetic energy and acceleration      (d) acceleration and momentum
2. A particle moves in a plane with a constant acceleration in a direction different from the initial velocity. The of the particle is  
(a) straight line      (b) arc of a circle      (c) parabola      (d) ellipse
3. A point moves with uniform acceleration and  $v_1, v_2$  and  $v_3$  denote the average velocities in the three successive intervals of time  $t_1, t_2$  &  $t_3$ . Which of the following relation is correct?  
(a)  $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 - t_2}{t_2 + t_3}$       (b)  $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 + t_2}{t_2 + t_3}$       (c)  $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 - t_2}{t_1 - t_3}$   
(d)  $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 - t_2}{t_2 - t_3}$
4. The initial velocity of a particle is  $u$  and the acceleration  $f$  is given by  $at$ . Which of the following relation is valid?  
(a)  $v = u + at^2$       (b)  $v = u + \frac{1}{2} at^2$       (c)  $v = u + at$   
(d)  $v = u$
5. A ball is released from the top of a tower of height  $h$  metre. If it takes  $T$  seconds to reach the ground, where is the ball at the time  $T/2$  s?  
(a) At  $(h/4)$  m from the ground      (b) At  $(h/2)$  m from the ground  
(c) At  $(3h/4)$  m from the ground      (d) Depends upon the mass of the ball
6. Two bodies one held 30 cm directly above the other are released simultaneously and fall freely under gravity. After 2 s, their relative separation will be  
(a) 10 cm      (b) 20 cm      (c) 30 cm      (d) zero
7. An automobile travelling with a speed of 60 km/h can brake to stop within a distance of 20 m. If the same vehicle is going twice as fast as previous speed, the stopping distance will be  
(a) 20 m      (b) 40 m      (c) 60 m      (d) 80 cm
8. A projectile can have the same range  $R$  for two angles of projection. If  $t_1$  and  $t_2$  be the time of flights in the two cases, then the product of the two times of flights is proportional to  
(a)  $R$       (b)  $1/R$       (c)  $1/R^2$       (d)  $R^2$
9. If two particles are projected with the same velocity. One vertically upwards and another vertically downwards from the top of a tower  
(a) they hit the ground simultaneously with different velocity  
(b) they hit the ground simultaneously with, with same velocity  
(c) they hit the ground at different instants, with different velocities

- (d) they hit the ground at different instants, with same velocities
10. The relation between time  $t$  and distance  $x$  is  $t = \alpha x^2 + \beta x$ , where  $\alpha$  and  $\beta$  are constants. The retardation is :
- (a)  $2 \alpha v^2$                       (b)  $2 \beta v^3$                       (c)  $2 \alpha \beta v^3$                       (d)  $2 \beta^2 v^3$
11. The position vector of a particle changes from 10 m east to 10 m north in one second. The magnitude of average velocity of the particle is
- (a) 20 m/s                      (b) 5 m/s                      (c)  $10\sqrt{2}$  m/s                      (d)  $5\sqrt{2}$  m/s
12. A person moves 30 m north, then 20 m east, then  $30\sqrt{2}$  m southwest. The displacement of the particle is
- (a) 14 m S-W                      (b) 28 m South                      (c) 10 m west  
(d) 15 m East
13. The direction of three forces 10N, 20N and 30N acting at a point are parallel to the sides of an equilateral triangle taken in order. The magnitude of their resultant is
- (a)  $5\sqrt{3}$  N                      (b)  $10\sqrt{3}$  N                      (c)  $20\sqrt{3}$  N                      (d) zero
14. A staircase contains three steps each having height 10 cm and width 20 cm. The minimum horizontal velocity of a ball moving off the upper marl plane so as to hit the lowest plane directly is
- (a) 4 m/s                      (b) 2 m/s                      (c) 1 m/s                      (d) 0.5 m/s
15. A particle is performing uniform circular motion with speed  $\pi$  m/s. If the radius of the circular path is 2 m, then magnitude of average velocity of the particle in the time interval 1.5 second is
- (a)  $\frac{2}{3}\sqrt{8+4\sqrt{2}}$                       (b)  $\frac{1}{3}\sqrt{4+4\sqrt{2}}$                       (c) zero                      (d)  $\frac{1}{3}\sqrt{2+2\sqrt{2}}$
16. An object with a constant speed
- (a) is not accelerated                      (b) might be accelerated  
(c) is always accelerated                      (d) also has a constant velocity
17. The distance travelled by a body falling from rest in the first, second and third seconds are in the ratio
- (a) 1:2:3                      (b) 1:3:5                      (c) 1:4:9                      (d) 1:1:1
18. An object covers distance in direct proportion to the square of the time elapsed. Its acceleration is
- (a) Increasing                      (b) decreasing                      (c) zero                      (d) constant
19. A body is projected vertically upwards. If  $t_1$  and  $t_2$  be the times at which it is at height  $h$  above the point of projection, while ascending and descending respectively, then height  $h$  is
- (a)  $\frac{1}{2} g t_1 t_2$                       (b)  $g t_1 t_2$                       (c)  $2 g t_1 t_2$                       (d)  $4 g t_1 t_2$

20. A particle starting from rest has a velocity that increases linearly with time as  $v = kt$ . Then the distance covered by it in first 3 s is
- (a)  $3k/2$                       (b)  $3k$                       (c)  $6k$                       (d)  $9k/2$

**ANSWER KEY**  
**KINEMATICS**

1	2	3	4	5	6	7	8	9	10
b	c	b	b	c	c	d	a	d	a
11	12	13	14	15	16	17	18	19	20
a	c	b	b	a	b	b	d	a	d